Enrollment No: ____________________ **Exam Seat No:____________________**

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name: Graph theory

Subject Code: 4SC06GTC1 **Branch:B.Sc.(Mathematics)**

Semester: 6 **Date :13/05/2016** **Time : 2:30 To 5:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Degree of pendant vertex is _________. **(01)**
(A) 3 (B) 2 (C) 1 (D) 0

b) An alternative sequence of vertices and edges in which no edge is covered more than once is called _________. **(01)**
(A) walk (B) circuit (C) self-loop (D) path

c) The degree of each vertex in complete graph K_n is **(01)**
(A) n -1 (B) n + 1 (C) 2n (D) n

d) A vertex with minimum eccentricity is called _________. **(01)**
(A) diameter (B) centre (C) radius (D) none

e) By removing cut-set from the given graph, it becomes _________ graph. **(01)**
(A) null (B) connected (C) disconnected (D) None

f) A graph is___________ if there is a path between any two of its vertices. **(01)**
(A) disconnected (B) closed (C) cycle (D) connected

g) Define: Parallel edges with illustration. **(02)**

h) Define: Spanning tree with illustration. **(02)**

i) Define: Fundamental Circuit **(02)**

j) Draw Petersen graph. **(02)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) State and prove first theorem of graph theory. Also prove that graph G must have even number of odd vertices. **(07)**

b) A graph G is disconnected if and only if its vertices set V can be partitioned into two non empty disjoint subsets V_1 and V_2 such that there exists no edges in G **(07)**

whose one end point in V_1 and another end point in V_2.

Q-3 **Attempt all questions** **(14)**

a) Let G be a simple graph with n vertices and k components. Then prove that G can have at most $\frac{(n-k)(n-k+1)}{2}$ number of edges. **(06)**

b) Let $G = (V, E)$ be a k −regular graph where k is an odd number then prove that number of edges in graph G is in multiple of k. **(06)**

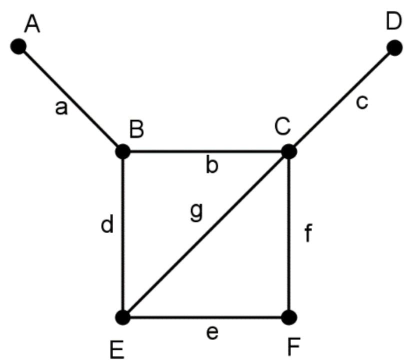
c) What is the smallest integer n such that the complete graph K_n has at least 500 edges? **(02)**

Q-4 **Attempt all questions** **(14)**

a) Explain Konisberg bridge problem.Solve it by using Euler's theorem. **(06)**

b) Let G be a connected graph. Then prove that G is an Euler graph if and only if each vertices in G is of even degree. **(06)**

c) Find a fusion graph of the following graph by fusing the vertices B and C. **(02)**



Q-5 **Attempt all questions** **(14)**

a) Let G be a tree with n vertices. Then prove that G has $(n-1)$edges. **(07)**

b) Let n be an odd number, $n \geq 3$. Then prove that there are exactly $\frac{n-1}{2}$ edge-disjoint Hamiltonian circuit in complete graph K_n. **(07)**

Q-6 **Attempt all questions** **(14)**

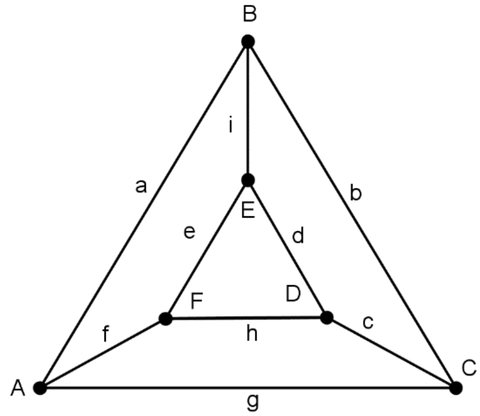
a) Without drawing graph check whether the graph corresponding to following adjacency matrix is connected or not. **(07)**

$$X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

b) Let G be an graph. Then prove that G is tree if and only if there exists a unique path between every pair of vertices in G. **(07)**

Q-7 **Attempt all questions** **(14)**

a) Find distance between every pair of vertices of G and eccentricity of every vertex. **(06)**

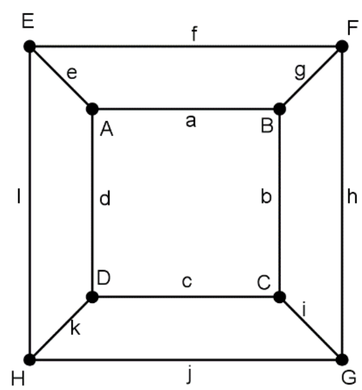


b) Prove that every tree has either one or two centers. **(06)**

c) Give two examples of tree with 7 vertices. **(02)**

Q-8 **Attempt all questions** **(14)**

a) Write adjacency and incidence matrices of the following graph: **(06)**



b) If the number of vertices is n in binary tree then prove that the number of pendant vertices is $\frac{n+1}{2}$. **(06)**

c) Define cut set and illustrate with graph. **(02)**